Study of the crossover transition of a gas of extended hadrons

L. Ferroni, V. Koch

Nuclear Science Division, Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley, 94720

Abstract

We formulate a simple model for a gas of extended hadrons at zero chemical potential by taking inspiration from the compressible bag model. We show that a crossover transition qualitatively similar to lattice QCD can be reproduced by such a system by including some appropriate additional dynamics. Under certain conditions, at high temperature, the system consists of a finite number of infinitely extended bags, which occupy the entire space. In this situation the system behaves as an ideal gas of quarks and gluons.

1. The gas of compressible hadrons

In this short note we will investigate a possible scenario for the crossover transition of a gas of hadrons at vanishing baryochemical potential (for a detailed description we suggest the reader to refer to [1]). Following the ideas proposed in [2], we adopt the philosophy that the same partition function should describe the system at both low and high temperature. To this end we describe hadrons as extended bags of Quark Gluon Plasma (QGP) in order to embody confined and deconfined phases from the very beginning. Accordingly (as derived in the simplest formulation of the MIT bag model [3]), we start by assuming a hadron mass spectrum of the Hagedorn's type $\rho(m) \propto \exp(m/T_0)/m^{\alpha}$. To model the transition, however, we need to add some additional dynamics. We found that a qualitatively good agreement with lattice QCD (LQCD) can be obtained by simply taking into account the elastic interactions between hadrons together with excluded volume corrections. The elastic interactions give rise to a *kinetic* pressure p_k that in turn tends to "squeeze" the bag-like hadrons, resulting in a generalized temperature-dependent mass spectrum. The stability condition for the existence of a bag is given by the pressure balance:

$$p_r = B + p_k(V,T) \,, \tag{1}$$

where p_r is the internal pressure of the bag, B is the bag constant and V and T are the system volume and temperature, respectively. In our simple scheme, we adopt Boltzmann statistics and a non-relativistic framework, which is a good approximation in the vicinity of the transition region [1].

In order to calculate the partition function of the system, one needs to evaluate the internal bags pressure p_r in Eq. (1) that depends on the kinetic pressure p_k. Because the pressure p_k is thermally generated, it must be calculated from the partition function itself, resulting in a self-consistency relation.

The behavior of the system depends on the value of the parameter α of the mass spectrum. For $\alpha > 5/2$ it exhibits a first order phase transition, whereas for $\alpha \leq 5/2$ the transition becomes a sharp crossover. Here, we analyze in detail the case of a crossover transition, i.e. $\alpha \leq 5/2$. As

an example we show the ratios $(\varepsilon - 3p_k)/T^4$ (Fig. (1a)) and $s/T^3 \equiv (\varepsilon + p_k)/T^4$ (Fig. (1b)), where ε and s are the energy and the entropy density, respectively. As one can see from (Fig. (1b)), for $\alpha = 0$ and $1/2$, the curves grow with the temperature with larger slopes for smaller α's. Instead, for $1 \leq \alpha \leq 5/2$, they settle onto constant asymptotic values in a qualitatively good agreement with LQCD. As α changes from 1 to 5/2 the asymptotic value converges very fast to the Stefan-Boltzmann limit from above [1]. In ref. [1], we also studied the ratios $p_k(\infty, T)/T^4$

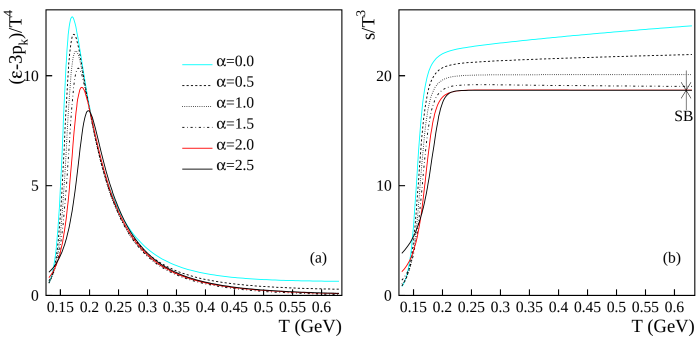

Figure 1: (Color online) Left panel (a): the ratio $(\varepsilon - 3p_k)/T^4$. Right panel (b): The ratio s/T^3, where s is the entropy-density.

and ε/T^4. Also these quantities exhibit a similar behavior. In the range $1 \leq \alpha \leq 5/2$, the model seems to produce a smooth crossover transition toward a new regime whose features are very similar to those of a gas of massless particles, even though no deconfined states are included in the partition function. We have also studied the (strong) dependence of the particle density $\langle n \rangle = \langle N \rangle / V$ (where $V \to \infty$) on α. We have found that there exist a limiting value α_0 between 2.12 and 2.13 such that for $\alpha_0 < \alpha \leq 5/2$ the particle density vanishes at high temperature. The system is then populated by one (or few) infinite bag(s) that occupies the entire volume. Conversely, for $\alpha < \alpha_0$, $\langle n \rangle$ grows with the temperature. In the range $1 \leq \alpha < \alpha_0$, the ideal gas behavior is mimicked by a number of heavy extended bags that saturate the phase space forming a dense system.

Acknowledgments

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[1] In this work we ignored the ~ 10% deviation of the LQCD result from the free gas (Stefan-Boltzmann) limit [4].